Stabilization of the Gain versus Frequency Characteristics of Parametric Amplifiers at High Input Signal Levels

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Abstract—**Decreasing the negative bias voltage of varactor diodes in a parametric amplifier causes the gain versus frequency characteristic of the amplifier to shift to the higher-frequency side, resulting in a so-called "positive slope" at the signal center frequency. The same happens when the pump power is increased or when the signal power is increased, but in the latter case only when the idler circuit load resistance is below a certain value.**

The slope of the gain characteristic can be partially or completely compensated by detuning the signal-circuit characteristic relative to the gain versus frequency characteristic in such a way that the latter is located on a certain point of the left or right slope of the signal-circuit characteristic, or by resistive loading of the idler circuit. Complete cancellation was achieved in the range from −30 to −20 dBm signal input power by using both methods simultaneously on a practical model of a parametric amplifier operating at a signal center frequency of 3.95 GHz and a pump frequency of 11.76 GHz. The loading of the idler circuit was done by drawing a little rectified diode current. The necessary increase in pump power, in order to maintain the same gain as with both signal and idler circuits tuned to resonance, was less than 3 dB, the increase in noise figure a few tenths of 1 dB from a typical value of approximately 3 dB.

Introduction

It is well known that a change in pump power level usually causes a severe slope in the gain versus frequency characteristics of a parametric amplifier. The gain peak moves toward the higher end of the signal frequency band when the pump power level is increased. This is due to the increase in average capacitance of the diode with increasing the pump power level, hence decreasing the resonant frequency of the idler circuit. To minimize this gain slope, a self bias is usually superimposed onto the fixed bias of the diode to cancel the change in average capacitance. This technique is successful in minimizing the effect of pump power variation on the gain slope. However, under large signal conditions (output power > −13 dBm), the average capacitance of the diode is also changed by the input signal level. Since the rectified diode current due to the large signal is much larger than that due to the pump for the same change in average capacitance, an optimum compensation bias resistance for pump power variation is too large for signal power variation. This results in overcompensation and causes opposite gain slope.

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Theoretical and Practical Considerations

This situation shall be investigated by looking into parametric amplifiers theory assuming abrupt junction varactors.[1] With the notation

Z_s = input impedance at the signal port
Z_p = input impedance at the pump port
R_s = bulk resistance of the varactor diode
$R_i + j\omega L_i = Z_i$ = load impedance of the idler circuit
$R_i = Re(Z_i)$
$R_{ser_{s,i,p}}$ = equivalent series loss resistance for signal, idler, and pump circuits
R_{par} = parallel loss resistance
$L_{s,i,p}$ = external tuning inductances for signal, idler, and pump circuits
S_0 = average elastance of the varactor
C_0 = average capacitance of the varactor
S_s, S_i, S_p = half amplitudes of the Fourier coefficients of the elastance for signal idler and pump frequency
S_{max}, S_{min} = reference values of the elastance
$m_0 = \dfrac{S_0}{S_{max} - S_{min}}$ normalized average elastance
$m_{s,i,p} = \dfrac{|S_{s,i,p}|}{S_{max} - S_{min}}$ signal, idler, and pump frequency modulation ratios
$\omega_c = \dfrac{S_{max} - S_{min}}{R_s}$ cutoff frequency of the varactor
$\omega_s = 2\pi \cdot$ signal frequency
$\omega_i = 2\pi \cdot$ idler frequency
$\omega_p = 2\pi \cdot$ pump frequency
V_0 = bias voltage
V_B = breakdown voltage
ϕ = contact potential
E_p = pump generator voltage.

The input impedance of the amplifier at the signal frequency is

[1] P. Penfield and R. P. Rafuse, *Varactor Applications.* Cambridge, Mass.: M.I.T. Press, 1962.

$$Z_s = R_s + j\omega_s L_s - j\frac{m_0\omega_c R_s}{\omega_s}$$

$$-\frac{m_p{}^2\omega_c{}^2}{\omega_s\omega_i}\frac{R_s{}^2}{R_s + Z_i{}^* + jm_0R_s(\omega_c/\omega_i)}. \quad (1)$$

The second and third terms in (1) represent the reactances of the resonant circuit for the signal frequency. At resonant frequency these terms cancel each other. The equivalent expression for the idler circuit is in the denominator of the fourth term. The bandwidth of the gain versus frequency characteristic is given by the bandwidth of the narrowest circuit in the amplifier, in our case of an unloaded idler circuit ($R_i = 0$) by the bandwidth of the idler circuit (narrow solid curve in Fig. 1).

If the bias voltage is decreased, the average elastance is decreased and the gain peak frequency shifts to the higher signal frequency side as shown by the narrow dotted curve and vice versa. The broad gain peak envelope represents roughly the resonance curve of the signal circuit; the narrow gain curve, the resonance curve of the idler circuit transformed into the signal frequency band. This transformation is represented by the fourth term in (1). Actually, a change in average elastance, caused by changing the bias voltage or the applied signal or pump power not only moves the narrow transformed idler resonance curve to the right or left frequency side, but also the broader signal resonance curve is shifted to the opposite side because the signal circuit is also detuned. So the envelope and the position and form of the gain curve are the product of *both* effects. For $\omega_i \gg \omega_s$ the amount for which the broad curve shifts for a certain change in average elastance is negligible in comparison to that of the transformed idler curve. To achieve maximum gain, usually signal and idler circuits are both tuned to resonance.

After having discussed the effect of changing the bias voltage, changes in signal and pump level need a closer investigation. Penfield and Rafuse,[1] assuming small signal theory and an abrupt junction, derived an expression for the change in average elastance for a variation ∂P in pump power. For a first approximation this should be sufficient. The result is

$$\partial m_0 = -\frac{m_p{}^2}{m_0}\frac{\partial P}{P}. \quad (2)$$

It indicates that increasing the pump power decreases the average elastance, i.e., an increase in pump power has the same effect as a decrease of bias voltage.

To investigate the effect of changes in signal level we have to look at the input impedance of the pump circuit, using large signal theory[1] of the abrupt junction

$$Z_p = R_s + j\omega_p L_p - j\frac{m_0\omega_c R_s}{\omega_p}$$

$$+\frac{m_s{}^2\omega_c{}^2}{\omega_p\omega_i}\frac{R_s{}^2}{R_s + Z_i - jm_0R_s(\omega_c/\omega_i)}. \quad (3)$$

For very low signal levels ($m_s \doteq 0$) and with the pump circuit tuned to resonance by means of L_p, $Z_p = R_s$. Therefore, the pump power has to be matched by a source impedance of R_s (Fig. 2). When the signal level and therefore m_s rise, Z_p increases as shown by the third term in (3). The first-order change in the real part of Z_p, when the idler circuit is resonant, is $(m_s{}^2\omega_c{}^2/\omega_p\omega_i)\,R_s{}^2/(R_s+R_i)$ and therefore the first-order change in pump modulation factor is

$$\frac{\partial m_p}{m_p} = -\frac{1}{2}\frac{\omega_c{}^2}{\omega_p\omega_i}\frac{R_s}{R_s + R_i}m_s{}^2. \quad (4)$$

From the fact that the Fourier coefficients of the elastance for abrupt junction varactors are directly proportional to their corresponding current components, the following relation can be derived for the three modulation factors[1]

$$m_i = m_s m_p \frac{\omega_c}{\omega_i}\frac{R_s}{|R_s + Z_i - jm_0R_s(\omega_c/\omega_i)|}. \quad (5)$$

On the other hand, the junction law for abrupt junction varactors yields the condition[1]

$$\frac{V_0 + \phi}{V_B + \phi} = m_0{}^2 + 2m_s{}^2 + 2m_i{}^2 + 2m_p{}^2. \quad (6)$$

Again assuming the idler circuit resonant and using (4)–(6), the expression for the change of the average elastance caused by a finite signal level becomes

$$\partial m_0 = -\frac{m_s{}^2}{m_0}\left[1 + \frac{m_p{}^2\omega_c{}^2}{\omega_i{}^2}\frac{R_s{}^2}{(R_s + R_i)^2} - \frac{m_p{}^2\omega_c{}^2}{\omega_p\omega_i}\frac{R_s}{R_s + R_i}\right]. \quad (7)$$

It is convenient to rewrite (7) by using the definitions for the modulation ratios and cutoff frequency into the form

$$\partial m_0 = -\frac{m_s{}^2}{m_0}\left[1 + \frac{|S_p|^2}{\omega_i{}^2(R_s+R_i)^2} - \frac{|S_p|^2}{\omega_p\omega_i R_s(R_s+R_i)}\right]. \quad (8)$$

For a lossless idler circuit ($R_i = 0$), the second term in the brackets is always larger than the third term; therefore, ∂m_0 is always negative for an increase in signal level as it is also always negative for an increase in pump power. On the other hand, if the idler circuit is lossy ($R_i \neq 0$), the third term decreases slower with increasing R_i than the second term. For a certain value of R_i which for practical values as

$$m_p = 0.15$$
$$\omega_c = 5 \cdot 10^{12}$$
$$R_s = 2\Omega$$
$$\omega_i = 2\pi \cdot 8 \cdot 10^9$$
$$\omega_p = 2\pi \cdot 12 \cdot 10^9$$

is in the order of magnitude as R_s, the three terms within the brackets cancel to zero. In this case the first-order change of the average elastance is zero for a variation in signal level from approximately zero to a level corresponding to the modulation factor m_s. A general conclusion which can be easily derived from (8) is: the smaller the ratio of ω_i/ω_s, the larger R_i has to be to set ∂m_0 to zero.

The foregoing considerations give us two possible ways for balancing the gain versus frequency characteristic of a parametric amplifier with respect to a change in signal level.

In the first case we assume that an increase in signal level creates a negative ∂m_0 where a detuning of the idler and signal resonance circuits causes a shift of the "transformed idler characteristic" to the higher end of the signal-circuit characteristic to the lower-frequency side. If both circuits were tuned to resonance as shown by the two solid curves in Fig. 1, this results in a so-called "positive slope" at the signal band center frequency f_{sc}.

If, on the other hand, the signal circuit had been detuned beforehand in such a way that the gain curve was located on the left or right slope of the envelope, partial or full cancellation of the *gain characteristic slope* for a certain range of signal levels is possible. The necessary condition is that the slope of the signal-circuit characteristic is not constant within the frequency range of the gain curve, but decreases at the higher-frequency side of the gain curve when the latter is located on the left slope of the signal-circuit characteristic, and increases when the gain curve is on its right side (Fig. 3).

In the first case, the signal and the transformed idler characteristics move somewhat together; in the second case, they move apart. In both cases, the effect of shifting the gain peak to the higher-frequency side is (at least partially) cancelled by the nonlinear characteristic of the signal-circuit characteristic. The amplitude is increased on the lower-frequency side of the gain curve and compressed on the higher-frequency side. In the ideal case (which only happens when the gain curve is located at the right spot of the signal-circuit characteristic and the latter has the right curvature) the gain peak stays at the same frequency when the signal level is varied within a certain range. There is only a more or less significant compression of the whole curve. Sometimes even overcompensation and a negative slope is possible.

This compensation procedure causes a decrease of gain. To obtain the same value as for simultaneous resonance of signal and idler circuit, the pump power has to be increased for a few (2–3) dB.

A second possibility to prevent a slope of the gain characteristic is by ohmic loading of the idler circuit to achieve $\partial m_0 = 0$. In this case neither the idler nor the signal circuit is detuned for an increase in signal level. This means that the gain characteristic stays flat at f_{sc}. For higher than optimum R_i values, positive values of ∂m_0 and overcompensation (negative slope) result.

These possibilities were investigated on a parametric amplifier with signal frequencies around 4 GHz and a pump frequency around 12 GHz. The 3 dB bandwidth of the envelope was about 500 MHz.

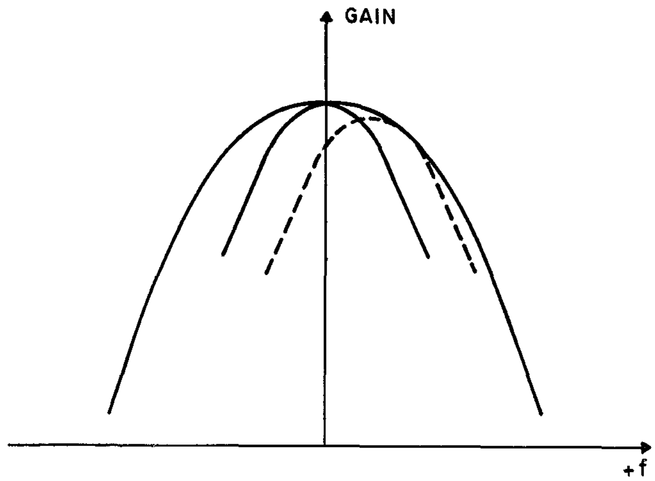

Fig. 1. Gain versus frequency characteristics and envelope.

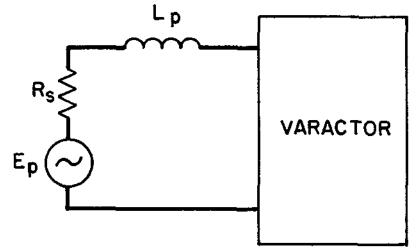

Fig. 2. Schematic equivalent pump circuit.

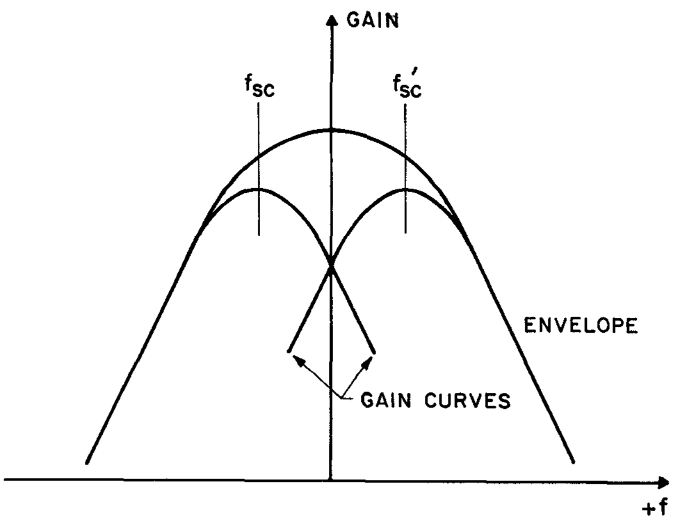

Fig. 3. Location of the gain versus frequency characteristics on the slopes of the signal-circuit characteristic for cancellation.

By detuning the signal circuit, complete cancellation and even overcompensation of the slope was possible when the gain curve was located on the right side of the signal-circuit characteristic. Figure 4 shows the gain versus frequency characteristics at −30 and −20 dBm signal input power. The idler and signal circuits were both tuned to resonance to obtain 12 dB gain at 3.95 GHz for −30 dBm input power. The pump power was 9 mW, the pump frequency 11.76 GHz. There is a noticeable slope at 3.95 GHz for −20 dBm input power. Figure 5 shows the situation with the signal circuit detuned in such a way that the gain characteristic was on its right slope. It was rather easy to find a position where complete cancellation occurred. The pump power for 12 dB gain was now 14 mW, the noise figure only a few tenths of a dB higher than before (typically around 3 dB).

With the gain curve located on the left slope of the signal-circuit characteristic, complete cancellation was not possible. On the other hand, it was found that in the cases when the gain characteristic was on the right slope, a little rectified diode current was drawn (typically about 0.6 and 0.8 μA for −30 and −20 dBm signal input power), but on the other

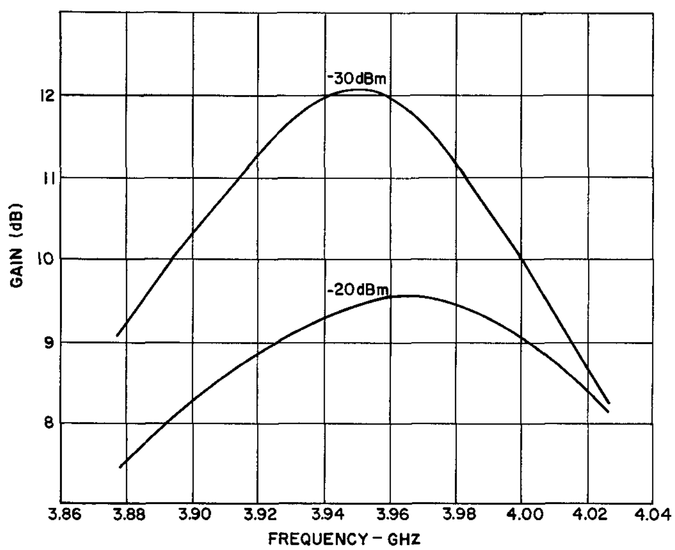

Fig. 4. Gain versus frequency characteristics for −30 and −20 dBm signal input power with both signal and idler circuits tuned for resonance.

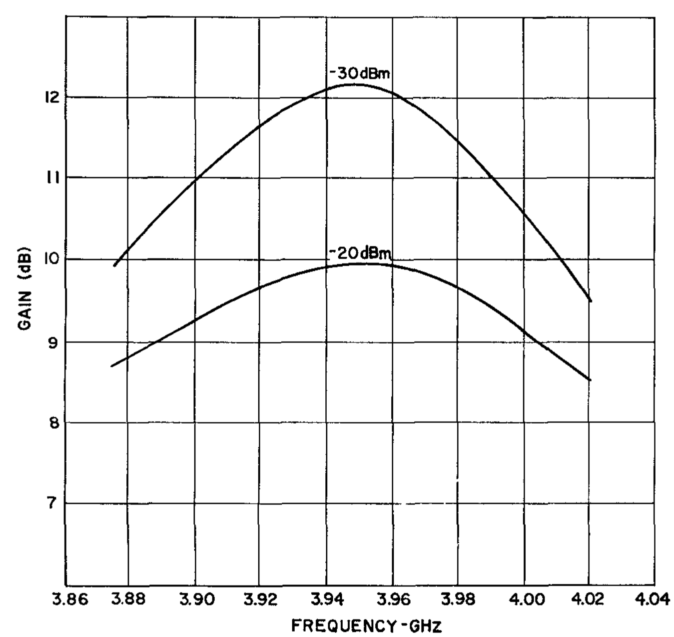

Fig. 5. Gain versus frequency characteristics for −30 and −20 dBm signal input power with the gain curve located on the right side slope of the signal-circuit characteristic and drawing rectified forward current for complete cancellation.

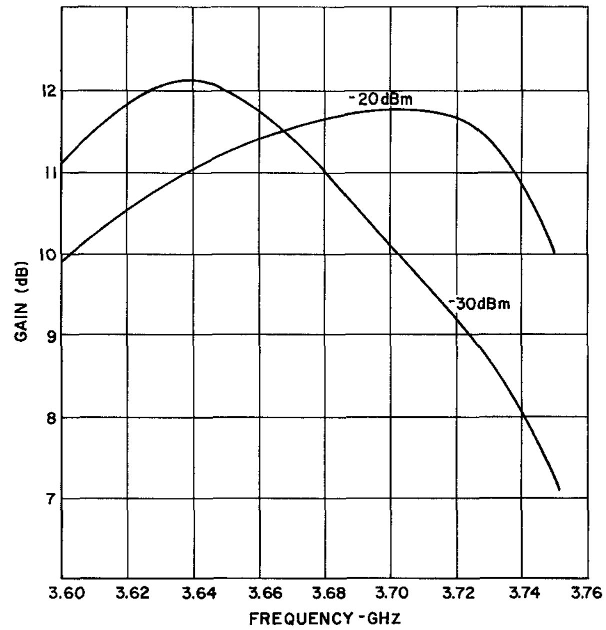

Fig. 6. Gain versus frequency characteristics for −30 and −20 dBm signal input power with the gain curve located at a point of the left side slope of the signal-circuit characteristic where normally no cancellation occurs.

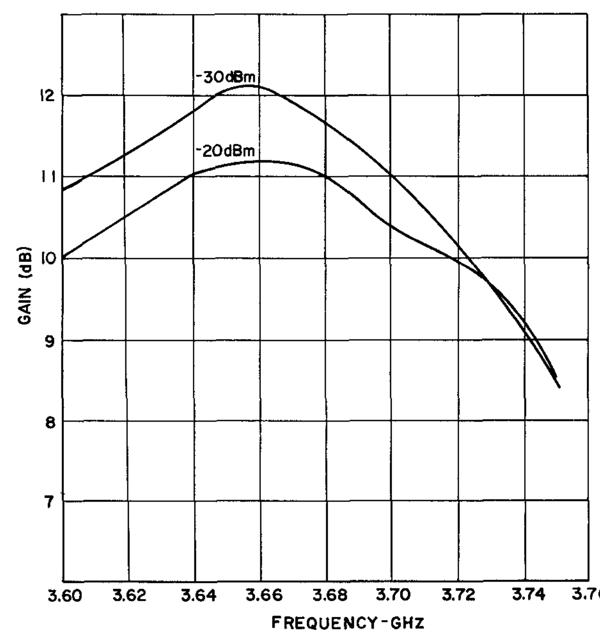

Fig. 7. Gain versus frequency characteristics for −30 and −20 dBm signal input power with the same situation as in Fig. 6 but with resistive loading for cancellation of the gain curve slope.

side the amplifier was always adjusted in such a way that the rectified current was only about 0.2–0.4 μA, respectively. From this it was concluded that the first method of cancellation was not sufficient with this amplifier. On the other hand, the rectified current is equivalent to a loss resistance in parallel to the diode junction. This parallel resistance R_{par} can be transformed into a frequency dependent series resistance R_{ser} which is

$$R_{\text{ser}} = \frac{R_{\text{par}}}{1 + R_{\text{par}}^2\omega^2C_0^2} \cdot \tag{9}$$

With these additional resistances in the signal idler and pump circuits, (8) has to be rewritten to

$$\partial m_0 = -\frac{m_s^2}{m_0}\left[1 + \frac{|S_p|^2}{\omega_i^2(R_s + R_{\text{ser}_i})^2} - \frac{2|S_p|^2}{\omega_p\omega_i(R_s + R_{\text{ser}_i})(2R_s + R_{\text{ser}_p})}\right]. \tag{10}$$

As $R_{\text{ser}_p} < R_{\text{ser}_i}$ there is again cancellation possible for a certain R_{par} and a certain ω_i/ω_p relationship. To prove the possibility of canceling the "positive gain slope" by resistive loading of the idler circuit, the amplifier was adjusted in such a way that the gain characteristic was located at a point of the left slope of the signal-circuit characteristic, where normally a strong positive slope occurred for a signal input power changing from −30 to −20 dBm (Fig. 6). The signal center frequency f_{sc} was this time 3.65 GHz, f_p = 11.76 GHz. The necessary pump power for 12 dB gain at −30 dBm input power was 10 mW. It was impossible with the amplifier tested to load the idler circuit without loading the two other circuits also. Therefore, a resistive graphite coating was put on the diode holders which introduced loss to all three circuits. As can be easily shown, again for a certain (now much higher than for single idler loading) value of loading resistance, cancellation should be possible. After tedious experimenting with the coating, cancellation was achieved as shown in Fig. 7. The pump power for 12 dB gain was 20 mW, the rectified diode current 0.2 and 0.3 μA for −30, and −20 dBm signal input power, respectively.

Conclusion

It was shown that balancing of parametric amplifiers in respect to changes in the input signal level is possible by detuning the signal circuit in a proper way relative to the gain versus frequency characteristic or by resistive loading of the idler circuit. In a practical example it was shown that the necessary increase in pump power was less than 3 dB, the increase in noise figure only a few tenths of a dB.

Acknowledgment

The author is indebted to K. Kurokawa for helpful discussions.

Correction

Y. Suematsu, K. Iga, and S. Ito, authors of "A Light Beam Waveguide Using Hyperbolic-Type Gas Lenses," which appeared on pages 657–665 of the December, 1966, issue of this TRANSACTIONS, have called the following to the attention of the Editor.

On page 661, (24) should be read:

$$w_{\text{in}} = w_0\Bigg[\left(\cos 2\phi - \frac{Q}{2}\sin 2\phi\right)\left\{Q\cosh 2\phi + \left(1 + \frac{Q^2}{4}\right)\sinh 2\phi\right\} + \left(\cosh 2\phi + \frac{Q}{2}\sinh 2\phi\right)\left\{Q\cos 2\phi + \left(1 - \frac{Q^2}{4}\right)\sin 2\phi\right\}\Bigg]^{1/2}$$
$$\times\left[1 - \left\{\cos 2\phi(\cosh 2\phi + Q\sinh 2\phi) - \sin 2\phi\left(Q\cosh 2\phi + \frac{Q^2}{2}\sinh 2\phi\right)\right\}^2\right]^{-1/4}$$

$$\frac{f_{\text{in}}}{k(0)} = 2w_0^2\Bigg[\left(\cos 2\phi - \frac{Q}{2}\sin 2\phi\right)\left\{Q\cosh 2\phi + \left(1 + \frac{Q^2}{4}\right)\sinh 2\phi\right\} + \left(\cosh 2\phi + \frac{Q}{2}\sinh 2\phi\right)\left\{Q\cos 2\phi + \left(1 - \frac{Q^2}{4}\right)\sin 2\phi\right\}\Bigg]$$
$$\times[\sin 2\phi(Q\cosh 2\phi + 2\sinh 2\phi) + Q\cos 2\phi\sinh 2\phi]^{-1}. \tag{24}$$

On page 663, in Table II, Length l_g should be expressed in the unit of meters instead of millimeters.